Notes on Consumer Theory:

Duality of Marshallian and Hicksian Demand Function

Theorem

Under regular assumption about utility function underlying the demand functions we have that:

1. $\boldsymbol{x_i^m(p,m) = x_i^h(p, v(p,m))}$
2. $\boldsymbol{x_i^h(p,\bar{u}) = x_i^m(p, e(p,\bar{u}))}$

Suppose $u(.) = x_1 x_2$ and income equal m; then we have:

$$x_1^m = \frac{m}{2p_1}$$

$$x_2^m = \frac{m}{2p_2}$$

Question: What will make these demand functions shift?

And we have:

$$v(p,m) = \frac{m}{2p_1}.\frac{m}{2p_2} = \frac{m^2}{4p_1p_2}$$

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Now suppose we fix utility at $\bar{u}$, then we have:

$$x_1^h = \left[\frac{\bar{u}p_2}{p_1}\right]^{\frac{1}{2}}$$

$$x_2^h = \left[\frac{\bar{u}p_1}{p_2}\right]^{\frac{1}{2}}$$

Question: What will make these demand functions shift?

And we will have:

$$e(p,\bar{u}) = p_1x_1 + p_2x_2 = p_1\left[\frac{\bar{u}p_2}{p_1}\right]^{\frac{1}{2}} + p_2\left[\frac{\bar{u}p_1}{p_2}\right]^{\frac{1}{2}}$$

$$e(p,\bar{u}) = 2[p_1p_2\bar{u}]^{\frac{1}{2}}$$

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Application of Theorem

Let us check:

2. $\boldsymbol{x_i^h(p,\bar{u}) = x_i^m(p,e(p,\bar{u}))}$

To do so suppose $\boldsymbol{m = 2[p_1p_2\bar{u}]^{\frac{1}{2}}}$

What will be the Marshallian demand for x_1?

$$\boldsymbol{x_1^m = \frac{m}{2p_1} \xrightarrow{plugging\ for\ m}}$$

$$\boldsymbol{x_1^m = \frac{2[p_1p_2\bar{u}]^{\frac{1}{2}}}{2p_1} = [p_1p_2\bar{u}]^{\frac{1}{2}}.p_1^{-1} = [p_2\bar{u}]^{\frac{1}{2}}.p_1^{-1}.p_1^{\frac{1}{2}} = [p_2\bar{u}]^{\frac{1}{2}}.p_1^{\frac{-1}{2}} = \left[\frac{\bar{u}p_2}{p_1}\right]^{\frac{1}{2}}}$$

And it is the Hicksian demand if $u(.) = \bar{u}$

So the theorem checked.

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You can check

1. $\boldsymbol{x_i^m(p,m) = x_i^h(p,v(p,m))}$

as an exercise.

Implications of Duality

I. Equality of Quantity Demanded

Duality means that given a set of prices, then the **quantity demanded** calculated using a Marshallian demand for a given level of income will be identical to the **quantity demanded** calculated using a Hicksian demand for the utility level that has been achieved by the choices made using the values obtained by Marshallian demands.

Alternatively, it says that given a set of prices, **the quantity demanded** calculated using a Hicksian demand for a fixed utility level will be identical to **the quantity demanded** calculated using a Marshallian demand if income is the minimum expenditure needed to remain at that fixed utility.

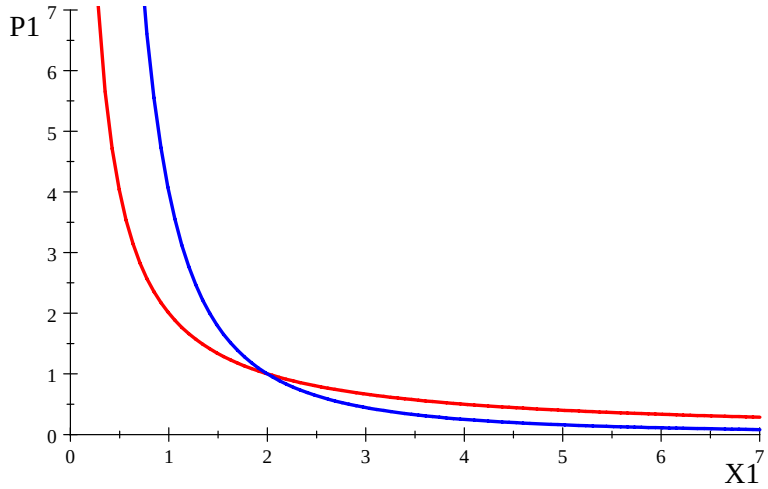
Put it simple, it says that Marshallian and Hicksian demand **have an intersection.** Moreover they intersect in **a consistent way.**

"Consistent way" means:

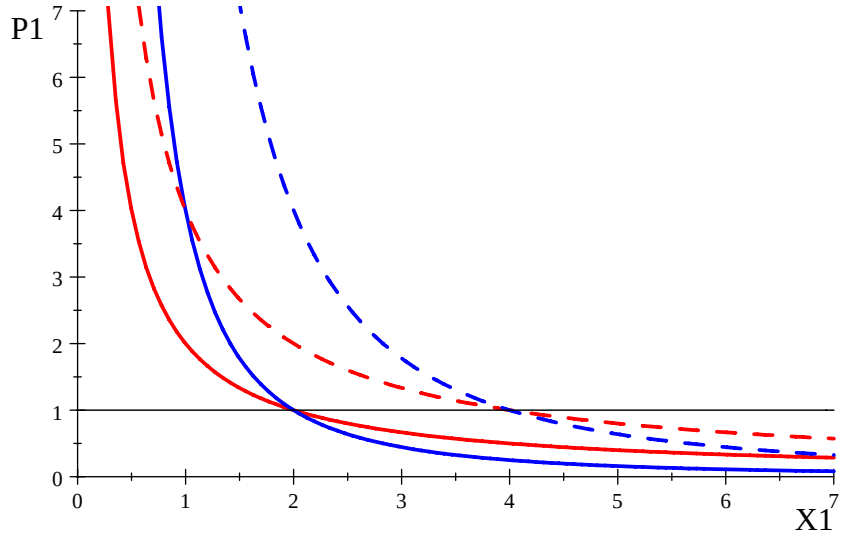
(i) Suppose income rises, increase in income in Marshallian conception makes utility increase and demand curve shift.

(ii) Now, if we shift a Hicksian demand by the magnitude of the change in utility in (i) the two demands intersect again at the same price level.

Red: Marshallian; Blue: Hicksian



After a consistent shift of both demands:



You see that they intersect at $p_1 = 1$ is the first place and then after a consistent shift.

II. For every quantity demanded computed using a Marshallian (Hicksian) demand there is a crossing Hicksian demand having as the parameter a different utility level (compensated income).

Optional: You can verify this property using the explicit functions given to you in this note, as an exercise.